EXTENDED VALUES FOR GEOMETRIC FACTOR OF EXTERNAL THERMAL RESISTANCE OF CABLES IN DUCT BANKS

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Abstract - In the Neher-McGrath analysis for calculating the external thermal resistance between cables in duct banks and ambient earth, approximate formulas for the geometric factor of the duct bank are employed which are valid only for a limited range of the height/width ratio of the cable bank. This limitation poses difficulties for power cable engineers when calculating external thermal resistances of some cable systems. This paper describes an efficient finite-element-based technique for calculating geometric factors for extended ranges of the height/width ratio and presents results for a wide range of values in a direct tabular and graphical format suitable for conventional Neher-McGrath calculations.

INTRODUCTION

In spite of the many developments of numerical techniques for power cable temperature rise and ampacity evaluation, the Neher-McGrath method [1] remains the technique most widely used by power cable engineers. This is mainly because of the simplicity of the method and its ability to provide reasonably accurate results for most practical cable systems in which complex configurations and conditions of soil, ambient and boundaries are not encountered. In addition, the use of the Neher-McGrath analysis was enhanced by several recent studies [2,3] which improve the accuracy of calculations and/or simplify implementation.

When calculating the geometric factor G_b [1] associated with the external thermal resistance between cable duct banks and ambient, the formulas derived [1,3] are based on approximating the rectangular duct bank by an isothermal circle of equivalent radius which is a function of the bank height and width dimensions. This approximation is valid only for height/width ratios in the range of 1/3 to 3. Therefore, the subsequent tabulated results (and graphs) of the geometric factor G_b are restricted to this range which may pose some difficulties to power cable engineers when calculating the external thermal resistance for cable banks with lower (or higher) height/width ratio. Although an accurate evaluation of the external thermal resistance can be performed using more detailed numerical techniques [4], the generalization of results in the form of charts or tables for extended ranges of cable bank dimensions would require numerous, time-consuming computations involving solutions of many sets of linear equations with full (non-sparse) coefficient matrices. In a recent study at Ontario Hydro, the efficient, sparsity-based finite-element solution and sensitivity analysis technique presented in [5] were utilized, in conjunction with appropriate analytical manipulations, to generate values of the geometric factor G_b for extended ranges of bank dimensions. This paper describes the analytical technique used and exhibits the results obtained in direct tabular and graphical forms which suit the conventional Neher-McGrath calculations. Dimensionless quantities are incorporated, whenever applicable, throughout the paper such that the analytical expressions and numerical results can be used with any consistent unit system.

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METHOD DESCRIPTION

The method described in this section is based on evaluating the external thermal resistance between a rectangular duct bank and an isothermal earth surface numerically for various duct bank dimensions and depths using an efficient finite-element scheme. Although the finite-element technique is general and can accommodate other heat transfer mechanisms (for example, constant heat flux generated inside the duct bank), the same assumptions as the conventional Neher-McGrath analysis were adopted. Specifically, the surface of the cable bank is assumed to be isothermal when calculating the geometric factor G_b.

Neher-McGrath Formula for G_b Factor

Using the IEC notation [3], which follows the Neher-McGrath analysis [1], a correction to the external thermal resistance of cable ducts (or pipes) given by

$$\Delta R = \frac{N}{2\pi} (\rho_e - \rho_c) \ln (u + \sqrt{u^2-1}) \tag{1}$$

is added algebraically to the thermal resistance expression to take account of the difference, if any, between the thermal resistivities of the cable bank material and the surrounding soil. In equation (1),

N = number of loaded cables in the bank
ρ_e = thermal resistivity of earth around bank
ρ_c = thermal resistivity of cable bank material
u = L_b/r_b
L_b = depth to center of duct bank
r_b = equivalent radius of bank given by

$$\ln r_b = \frac{1}{2} \frac{x}{y} \left(\frac{4}{\pi} - \frac{x}{y}\right) \ln \left(1 + \frac{y^2}{x^2}\right) + \ln \frac{x}{2} , \tag{2}$$

where x and y denote, respectively, the shorter and longer sides of the bank section.

The geometric factor G_b is given by the last term of equation (1), namely

$$G_b \triangleq \ln (u + \sqrt{u^2-1}) . \tag{3}$$

As described before, the approximation of the duct bank surface as an isothermal circle will restrict [1,3] the application of the above formulas to the range of $y/x \leq 3$.

Finite-Element Extended Analysis for G_b Factor

Consider the thermal circuit configuration of Figure 1 where the cable bank is represented by the rectangular isothermal surface C of height h and

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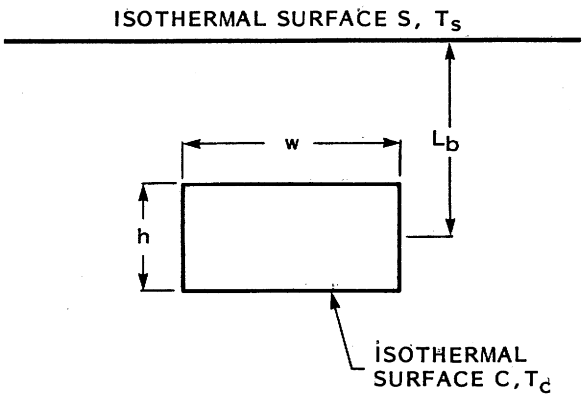

Fig. 1. Thermal circuit configuration

width w and of temperature T_c. For the configuration of Figure 1, the external thermal resistance between the bank surface C and the isothermal earth surface S, per unit length, is given by [6]

$$R = \rho(T_s - T_c)/\int_C (\partial T/\partial n)ds, \tag{4}$$

where the integration is performed along the bank surface C, $\partial/\partial n$ denotes differentiation along the normal to C, ρ is the thermal resistivity of the medium.

The computational technique presented in this paper is based on solving the boundary value problem of Figure 1 using an efficient finite-element scheme and then using a discretized form of equation (4) to evaluate the external thermal resistance R and consequently the geometric factor G_b. In the finite-element solution, the medium surrounding the surface C of Figure 1 is partitioned into small triangles constituting a finite-element grid such that the first grid layer, enclosing the bank surface C, is carefully structured, as shown in Figure 2, to attain an efficient subsequent evaluation of (4). The temperature of the isothermal earth's surface S is set to 0°C in the finite-element analysis while the temperature of the isothermal surface C is set to 1°C. The surface C is partitioned into K small segements, as shown in Figure 2, where the temperatures T_1, T_2, T_3,... of the middle points of the first grid layer (which constitute nodes of the finite-element grid) are evaluated. The accuracy of the solution can be controlled by adjusting the size of the finite-elements of the grid. Equation (4) can then be written in the discretized form

$$R = -\rho/\sum_{i=1}^{K}\left(\frac{\Delta T_i}{\Delta n_i}\Delta S_i\right) \quad , \tag{5}$$

where, $\Delta T_i = T_i - T_c = T_i - 1$, ΔS_i denotes the width of segment i (along the surface C) and Δn_i denotes the height of the grid layer at segment i. Note that the quantitites $\Delta T_i/\Delta n_i$ approximate the gradient $\partial T/\partial n$ along the surface C. By choosing $\Delta S_i/\Delta n_i = 1$ for all i = 1, ..., K, equation (4) reduces to

$$R = -\rho/\left(\sum_{i=1}^{K} T_i - K\right) . \tag{6}$$

Therefore, once the finite-element solution is obtained, the external thermal resistance can be evaluated simply by summing the temperature T_1, T_2, ..., T_K along the grid layer #1 and substituting in (6). The geometric factor G_b defined as $2\pi R/\rho$ is therefore given by

$$G_b = 2\pi/\left(K - \sum_{i=1}^{K} T_i\right) . \tag{7}$$

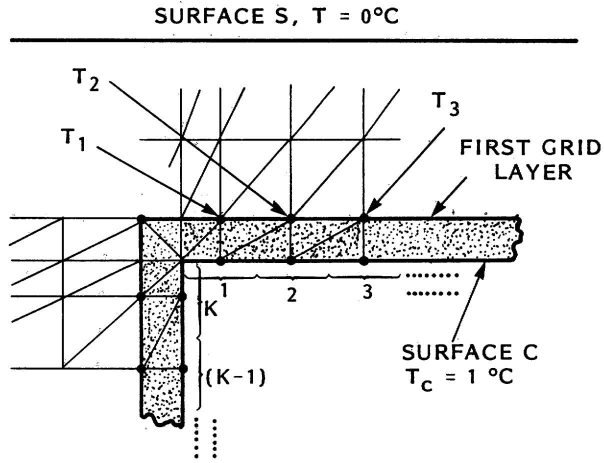

Fig. 2. Finite-element grid structure

Unlike the application of equation (3) [1,3], the geometric factor G_b as given by equation (7) can be calculated for any height/width ratio of the cable bank as well as for any bank depth. The values of G_b presented in the next section for extended height/width ratios have been generated by repeating the finite-element analysis for various ranges of bank height h, width w and depth L_b and substituting the resulting temperatures in equation (7). Because of the numerous finite-element analyses required to generate values of G_b for various ranges of dimensional parameters, an extremely efficient finite-element computational scheme is needed to cope with the increased computational burden. Although the mathematical details of the decomposed, sparsity-based finite-element technique employed are beyond the scope of the paper, the following points are to be noted:

1. The finite-element algorithm exploits the special sparsity form associated with the grid structure of Figure 2 to attain minimum storage and computational time per finite-element analysis.

2. Symmetry of the thermal circuit configuration is exploited to further reduce the computational time and storage by approximately 50%.

3. Because only the temperatures at the grid nodes immediately surrounding the bank surface C are needed in the calculations, the computerized scheme is carefully structured to solve only for these temperatures leading to further saving in computational time.

4. Only one finite-element grid formulation is needed. Variations of cable bank dimensions and/or depth are simulated simply by adjusting a few scaling parameters which control the relative size of the grid elements.

5. The accuracy of the solution and its sensitivities with respect to variations in thermal circuit parameters are evaluated and examined in an efficient way using the sensitivity approach described in [5].

TABLE I

EXTENDED VALUES OF GEOMETRIC FACTOR G_b FOR DUCT BANKS

h/W \ L_b/h	0.6	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	20.0
0.05	0.08	0.32	0.39	0.59	0.77	0.93	1.08	1.21	1.34	1.45	1.56	1.67	1.77	1.87	1.96	2.05	2.14	2.23	2.31	2.47
0.1	0.10	0.36	0.65	0.94	1.18	1.39	1.57	1.72	1.87	2.00	2.13	2.25	2.37	2.47	2.57	2.66	2.76	2.85	2.94	3.12
0.2	1.14	0.45	1.00	1.37	1.68	1.93	2.12	2.24	2.39	2.53	2.66	2.79	2.90	3.01	3.12	3.21	3.31	3.41	3.51	3.69
0.3	0.18	0.56	1.26	1.68	2.02	2.29	2.48	2.60	2.75	2.89	3.02	3.15	3.27	3.38	3.49	3.59	3.69	3.80	3.89	4.08
0.4	0.22	0.68	1.43	1.86	2.19	2.45	2.66	2.80	2.95	3.09	3.22	3.35	3.47	3.58	3.69	3.79	3.88	3.95	4.02	4.12
0.5	0.25	0.81	1.51	1.92	2.21	2.46	2.67	2.83	2.99	3.13	3.25	3.38	3.50	3.61	3.71	3.81	3.91	4.01	4.11	4.29
0.6	0.29	0.90	1.62	2.04	2.34	2.59	2.81	2.98	3.15	3.29	3.42	3.55	3.68	3.80	3.91	4.02	4.13	4.24	4.35	4.56
0.7	0.32	0.97	1.71	2.14	2.44	2.70	2.92	3.10	3.27	3.43	3.57	3.72	3.86	3.99	4.12	4.24	4.37	4.49	4.62	4.86
0.8	0.35	1.04	1.81	2.26	2.58	2.87	3.12	3.34	3.55	3.74	3.92	4.11	4.29	4.47	4.64	4.81	5.00	5.19	5.39	5.79
0.9	0.39	1.11	1.90	2.39	2.74	3.07	3.37	3.64	3.91	4.16	4.40	4.65	4.90	5.15	5.39	5.63	5.89	6.14	6.41	6.94
1.0	0.42	1.17	2.00	2.52	2.93	3.31	3.67	4.01	4.35	4.68	5.01	5.34	5.68	6.01	6.35	6.68	7.01	7.34	7.67	8.33
1.2	0.47	1.24	2.06	2.58	2.98	3.35	3.70	4.03	4.36	4.68	5.02	5.34	5.67	5.98	6.30	6.61	6.93	7.25	7.57	8.21
1.4	0.52	1.31	2.12	2.64	3.03	3.40	3.75	4.08	4.41	4.73	5.05	5.37	5.69	6.00	6.31	6.62	6.92	7.25	7.57	8.20
1.6	0.56	1.37	2.18	2.70	3.10	3.47	3.82	4.15	4.48	4.81	5.14	5.46	5.78	6.09	6.40	6.71	7.03	7.34	7.66	8.29
1.8	0.60	1.43	2.24	2.76	3.17	3.55	3.91	4.24	4.58	4.92	5.26	5.59	5.92	6.24	6.56	6.87	7.19	7.52	7.85	8.50
2.0	0.64	1.48	2.31	2.83	3.25	3.64	4.01	4.36	4.72	5.07	5.43	5.78	6.12	6.45	6.78	7.11	7.45	7.79	8.13	8.82
2.2	0.67	1.52	2.39	2.90	3.35	3.77	4.17	4.55	4.94	5.32	5.71	6.09	6.47	6.84	7.21	7.58	7.96	8.33	8.71	9.46
2.4	0.70	1.56	2.46	2.98	3.44	3.89	4.32	4.74	5.16	5.58	6.00	6.42	6.83	7.24	7.65	8.05	8.46	8.87	9.28	10.11
2.6	0.73	1.59	2.53	3.05	3.54	4.02	4.49	4.94	5.39	5.84	6.29	6.74	7.19	7.63	8.08	8.52	8.97	9.41	9.86	10.75
2.8	0.76	1.62	2.60	3.13	3.65	4.15	4.65	5.13	5.62	6.10	6.58	7.06	7.55	8.03	8.51	8.99	9.47	9.96	10.44	11.41
3.0	0.79	1.64	2.66	3.20	3.74	4.28	4.81	5.33	5.85	6.37	6.88	7.40	7.92	8.43	8.95	9.47	9.99	10.51	11.02	12.06
3.2	0.82	1.67	2.72	3.27	3.84	4.41	4.97	5.53	6.08	6.63	7.18	7.73	8.29	8.84	9.39	9.95	10.50	11.06	11.61	12.72
3.4	0.84	1.70	2.77	3.35	3.95	4.55	5.14	5.73	6.32	6.90	7.48	8.07	8.66	9.25	9.84	10.43	11.02	11.61	12.20	13.38
3.6	0.86	1.72	2.81	3.42	4.05	4.68	5.31	5.94	6.56	7.10	7.79	8.41	9.04	9.66	10.29	10.92	11.54	12.17	12.79	14.04
3.8	0.88	1.75	2.85	3.49	4.16	4.82	5.48	5.14	6.80	7.45	8.10	8.76	9.42	10.08	10.74	11.41	12.07	12.73	13.39	14.71
4.0	0.90	1.77	2.89	3.56	4.26	4.96	5.66	6.35	7.04	7.73	8.42	9.11	9.81	10.50	11.20	11.90	12.60	13.29	13.99	15.38
4.5	0.94	1.83	2.96	3.74	4.53	5.31	6.10	6.88	7.66	8.44	9.22	10.00	10.79	11.57	12.35	13.14	13.93	14.71	15.50	17.08
5.0	0.97	1.88	3.00	3.91	4.79	5.67	6.55	7.42	8.29	9.17	10.04	10.91	11.79	12.66	13.53	14.40	15.28	16.15	17.03	18.79

RESULTS AND DISCUSSION

The finite-element-based technique described in the previous section to evaluate the geometric factor G_b has been implemented to generate values of G_b for an extended range of cable bank height/width ratio. The calculations have been performed on a UNIVAC-1100 computer at Ontario Hydro using an extended version of the finite-element sensitivity analysis algorithm described in [5]. The results obtained are shown in Table I and drafted in Figure 3. For the purpose of comparison, similar results obtained by implementing the conventional formula (3) are also shown in Table II. Note that the results of Table I are displayed in terms of the height/width (h/w) and depth/height (L_b/h) ratios rather than the shorter/longer side (x/y) and depth/perimeter (L_b/P) as used in [1].

TABLE II

VALUES OF G_b AS CALCULATED FROM EQUATION 3

y/x \ L_b/x	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
1.0	1.21	1.97	2.38	2.67	2.90	3.08	3.24	3.37
1.2	1.12	1.89	2.31	2.60	2.83	3.01	3.17	3.30
1.4	1.06	1.84	2.26	2.55	2.78	2.96	3.11	3.25
1.6	1.00	1.80	2.20	2.51	2.73	2.92	3.08	3.21
1.8	0.96	1.76	2.18	2.48	2.70	2.89	3.04	3.18
2.0	0.93	1.74	2.16	2.45	2.68	2.86	3.02	3.15
2.2	0.91	1.72	2.14	2.44	2.66	2.85	3.00	3.14
2.4	0.89	1.71	2.13	2.42	2.65	2.83	2.99	3.12
2.6	0.88	1.70	2.12	2.41	2.64	2.82	2.98	3.11
2.8	0.87	1.69	2.11	2.41	2.63	2.82	2.97	3.11
3.0	0.86	1.69	2.11	2.40	2.63	2.81	2.97	3.10

Remarks

The results shown in Tables I and II, indicate that, for a given value of L_b/h, the G_b factor calculated using the Neher-McGrath method (Table II) tends to decrease as the ratio h/w increases, in the range of $h/w > 1.0$, which contradicts the more exact results obtained by the finite-element scheme (Table I). This contradiction is due to the fact that the Neher-McGrath method approximates the rectangular duct bank by an isothermal circle (as shown in Figure 4a) and therefore it does not differentiate (in the computation) between cases where $h > w$ and cases where $h < w$. On the other hand, the finite-element scheme recognizes the difference betwen the heat flux patterns associated with these two cases as shown in Figure 4b.

For the special case of $h/w = 1$, the results of Table II agree within a few percent with the Goldenberg analytical formula [6]

$$G_b = \ln\left(1.6944\,\frac{2L_b}{h}\right) \tag{8}$$

for values of L_b/h in the range of 1.0 to 3.0. The empirical formula (8) generates more conservative values of G_b for $L_b/h < 1.0$ and less conservative values of G_b for $L_b/h > 1.0$ in comparison with the finite-element results. Note that formula (8) is applicable only to the case where $h = w$.

CONCLUSIONS

Values of the geometric factor G_b, used in the conventional Neher-McGrath analysis to calculate the external thermal resistance of cable banks, have been calculated and displayed for extended ranges of the height/width ratio. Values of G_b have been generated

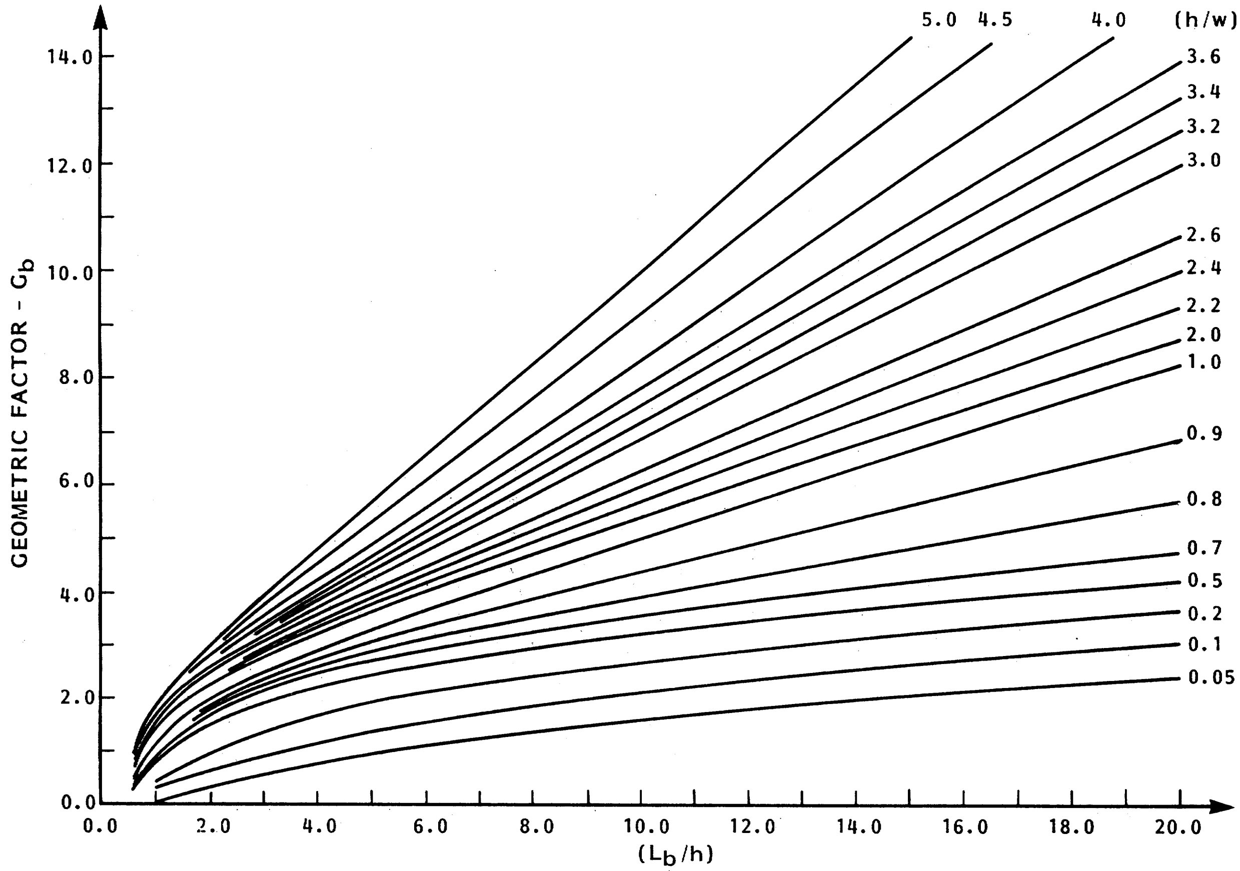

Fig. 3. Values of geometric factor G_b for duct banks

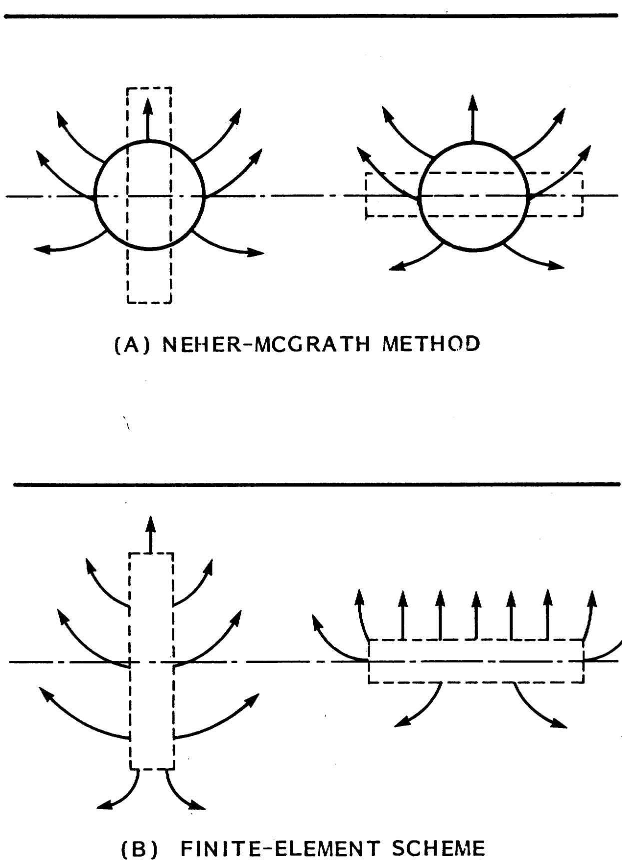

Fig. 4. Comparison between Neher-McGrath and proposed method

using an efficient, accurate finite-element scheme which facilitated the numerous finite-element solutions required to generate these values. The results obtained for the case where h/w = 1.0 agree to within a few percent with values calculated using the Neher-McGrath method. However, for all other values of h/w, the results obtained differ markedly from the values calculated using the Neher-McGrath method, and, indeed, indicate that the two cases h > w and h < w cannot be treated identically.

The values of the geometric factor G_b reported in this paper are intended to provide power cable engineers with accurate values over a wide range of L_b/h and h/w suitable for direct use in the conventional Neher-McGrath analysis. Although the duct bank surface was assumed, in the present analysis, to be an isothermal surface as in the conventional Neher-McGrath analysis, the finite-element technique is general and can accommodate other heat transfer mechanisms inside the duct bank. In that case, however, the results obtained would suit only a particular class of cable configurations and could not be generalized.

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Discussion

N. R. Spencer and **G. A. MacPhail** (BC Hydro and Power Authority, Vancouver, British Columbia, Canada): This paper makes a very significant contribution towards obtaining more precise values of ampacity for cables in ductbanks and is particularly appropriate at this time when increased confidence in the load capability of existing cable circuits may be used to postpone or defer capital expenditures for augementation or replacement.

Although the paper refines the Neher McGrath method of calculating ampacities and will certainly be adopted by many utilities, it still assumes that the ductbank/earth interface is isothermal and that the surrounding media has a uniform value of thermal resistivity. These assumptions are not necessarily true. What is needed now is a method of calculating ampacities taking into account the thermal gradients through the ductbank and variations in thermal resistivity of the surrounding media.

Have the authors investigated the validity of the assumption of an isothermal ductbank surface for extended height/width ratios? How sensitive are the extended values of the geometric factor G_b to the assumptions of an isothermal ductbank surface and homogenous external thermal resistance?

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M. A. El-Kady and **D. J. Horrocks:** We would like to thank Messrs. Spencer and MacPhail for thier valuable and pertinent comments.

As the discussers noted, the Neher McGrath method of calculating cable ampacities assumes an isothermal duct bank surface and uniform thermal resistivity in the surrounding soil. We adopted the same assumptions in calculating the extended values of the G_b factor so that cable engineers could continue using existing calculation methods. We believe that, given those assumptions, cable ampacities may be calculated more accurately using the G_b values presented in our paper than was possible before.

The duct bank surface is not an isothermal, although that assumption is probably reasonable for a square, or close to square, duct bank buried fairly deeply. As the value of the (H/W) ratio departs from unity and/or the value of the ratio (L_b/H) gets smaller, the assumption is less and less valid. The cable engineer should be aware of this limitation of the Neher McGrath method and use the results of such calculations with the caution appropriate to the assumptions and data used to obtain them.

Soil thermal resistivity in any situation is rarely uniform even without the effect of the heat from buried cables. However, in most cases, it is reasonable to assume uniform thermal resistivity for the soil outside the duct bank or cable backfill, especially if the latter has been designed to take account of the drying effect of the heat due to cable losses. We have calculated cable ampacities of cables in complex backfill arrangements which were not amenable to solutions by conventional methods; those calculations required the use of finite element techniques (1,2).

We agree that there is a need for a calculation method, which takes account of the variation of thermal resistivity in the media surrounding a duct bank and the fact that the duct bank surface is not an isothermal. Allowing for thermal resisitivity variations would likely result in a more complicated method than that of Neher McGrath, the simplicity of which accounts, among other things, for its continued use. On the other hand, the problem of dealing with the mechanism of heat transfer from the duct bank is one which, we believe, can be handled by extensions to existing methods. We consider that the G_b factor must take account of the heat flux through the duct bank surfaces, which is a much more complicated condition to model than the isothermal surface. Such a method would allow for the number of cables in the duct bank and their positions relative to each other and the duct walls, as well as the duct bank dimensions and depth of bury. However, via the definition of appropriate configuraiton indices and extensive "off-line" simulations, a final set of generalized, refined values of the G_b factor can be derived and displayed in a compact tabular or graphical form. It is our intention to develop a technique to calculate G_b factors on that basis.

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